

Birkhoff Decompositions and Photonic Interconnects

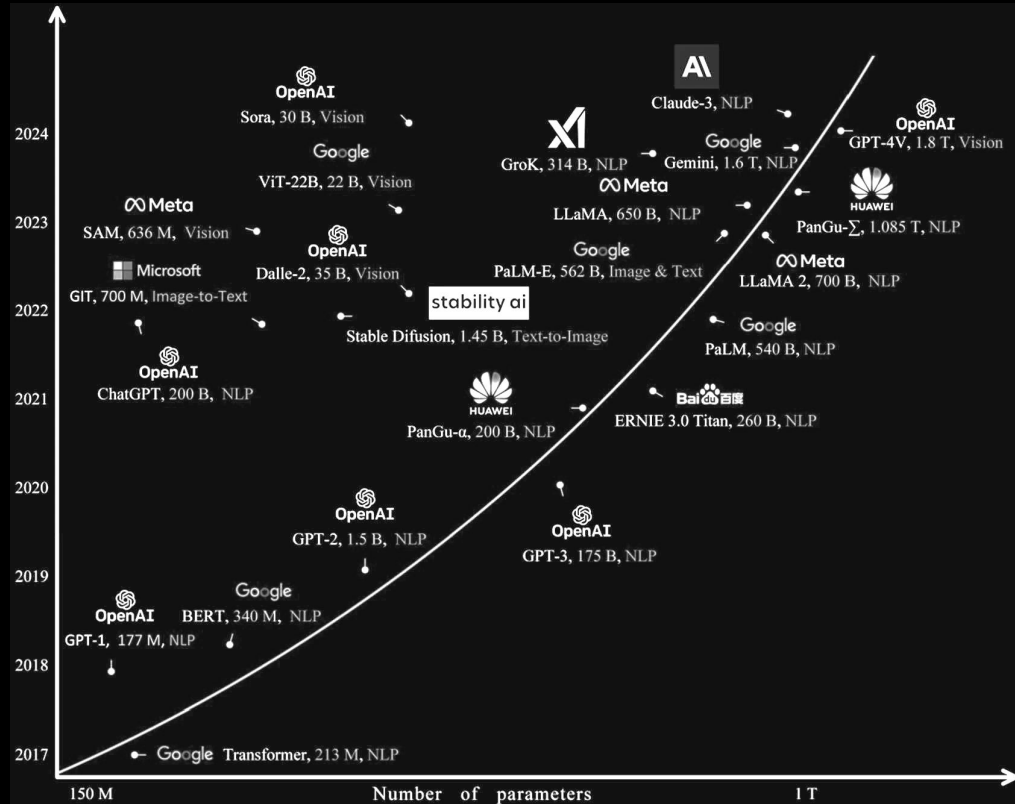
Wait! Don't Forget the Compute!

Eliezer Amponsah, Vamsi Addanki



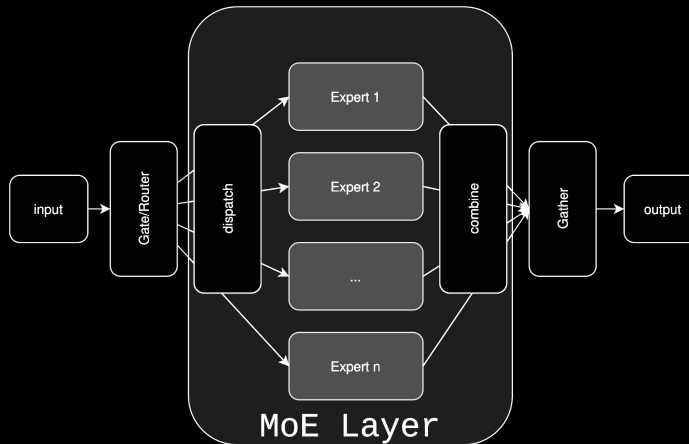
Emerging Trends: Increasing Size of AI Models

- Increasing size of AI models and workloads in DCN



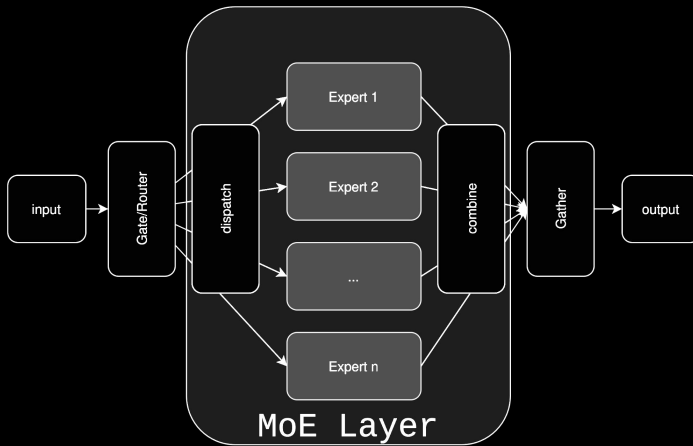
Emerging Trends: Increasing Size of AI Models

- MoE Innovation



Emerging Trends: Increasing Size of AI Models

- MoE Innovation
- Sparse Activations
 - DeepSeek-V3: 671B - **37B** activated
 - Qwen 3.5: 397B - **17B** activated
 - Mixtral 8x22B: 141B - **39B** activated
- Gate/Router



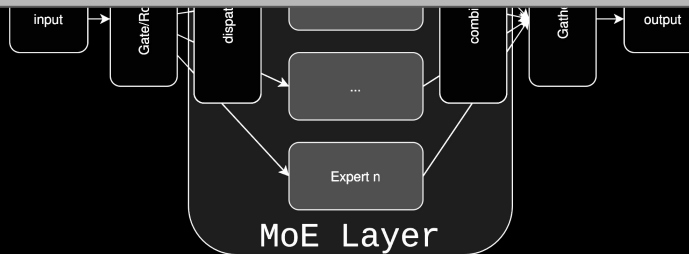
Emerging Trends: Increasing Size of AI Models

- MoE Innovation

- Sp

Compute no longer scales with model size

- G



The Rise of Dynamic Communication Patterns

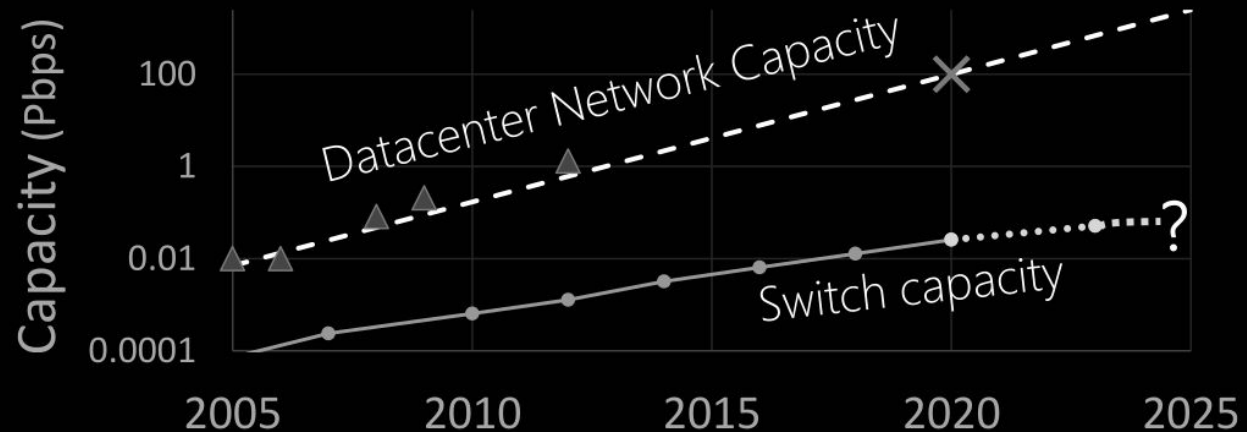
- All-to-All, unlike allreduce that allows static ring
- All to all structure changes often!

The Rise of Dynamic Communication Patterns

- All-to-All, unlike allreduce that allows static ring
- All to all structure changes often!
- Exacerbated in MoE
 - Varies on each input
 - Each layer

The Need for Silicon Photonic Interconnects

- Electrical interconnects are facing saturation with the near-end of moore's law...



Ballani, Hitesh, et al. "Sirius: A flat datacenter network with nanosecond optical switching."

Emerging Challenge

- **Communication is dynamic**
 - One topology cannot rule them all
 - Reconfiguration becomes necessary!
- **Photonic Interconnects are circuit switched**
 - **Requires reconfiguration to satisfy all-to-all communication**

Traditional Approach: Birkhoff Decomposition

	1	2	3	4	5	6	7	8
1								
2								
3								
4								
5								
6								
7								
8								

Row = sender

Column = receiver

$D_{i,j}$ = src i sends to recvr j

Demand Matrix

Traditional Approach: Birkhoff Decomposition

	1	2	3	4	5	6	7	8
1								
2								
3								
4								
5								
6								
7								
8								

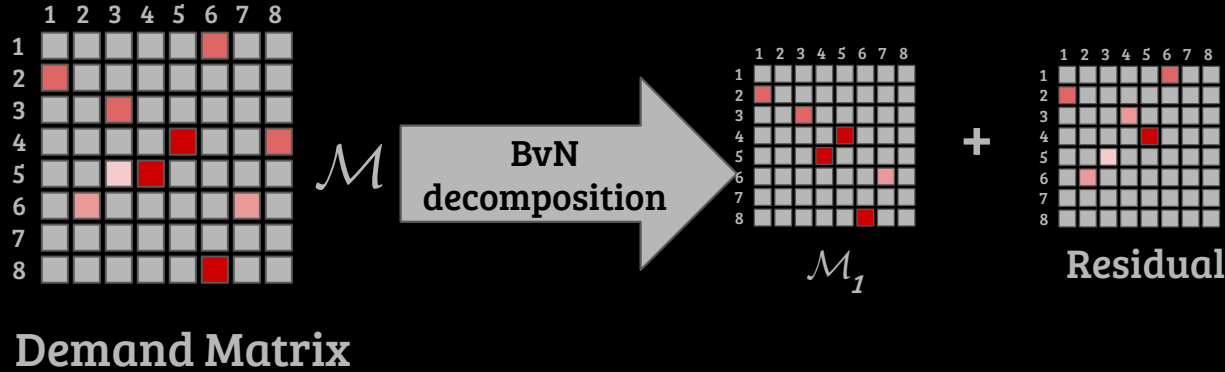
$\mathcal{M}$

BvN
decomposition

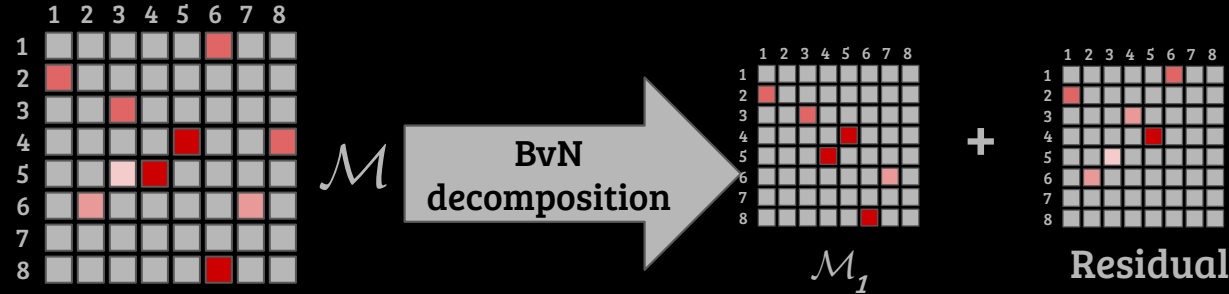
Find the minimum non-zero entry and
extract a permutation (matching)

Demand Matrix

Traditional Approach: Birkhoff Decomposition



Traditional Approach: Birkhoff Decomposition



Demand Matrix

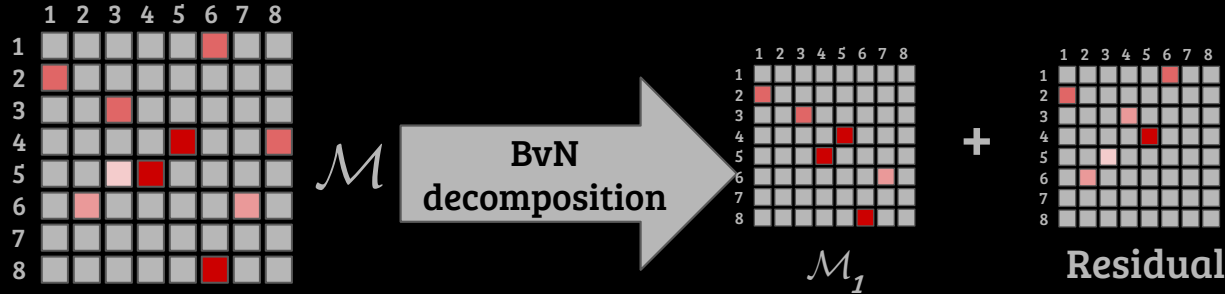


A requirement for Bvn

Doubly stochastic

All rows and column sum to the same value

Traditional Approach: Birkhoff Decomposition



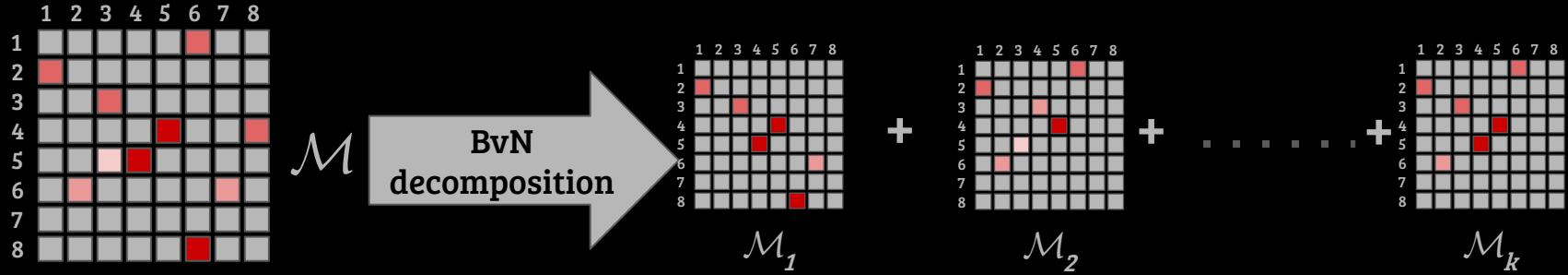
Demand Matrix

A property of Bvn

Doubly stochastic

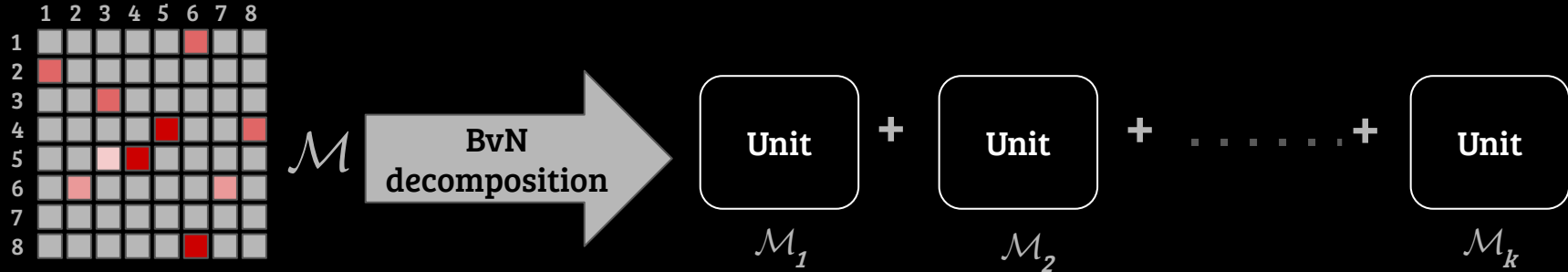
All rows and column sum to the same value

Traditional Approach: Birkhoff Decomposition



Demand Matrix

Traditional Approach: Birkhoff Decomposition



Demand Matrix

Traditional Approach: Birkhoff Decomposition

	1	2	3	4	5	6	7	8
1								
2								
3								
4								
5								
6								
7								
8								

Demand Matrix

$\mathcal{M}$

BvN
decomposition

Unit

$\mathcal{M}_1$

+

Unit

$\mathcal{M}_2$

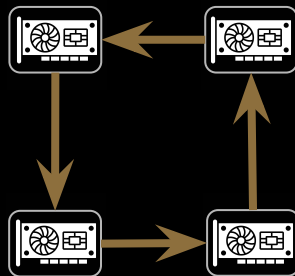
+

...

+

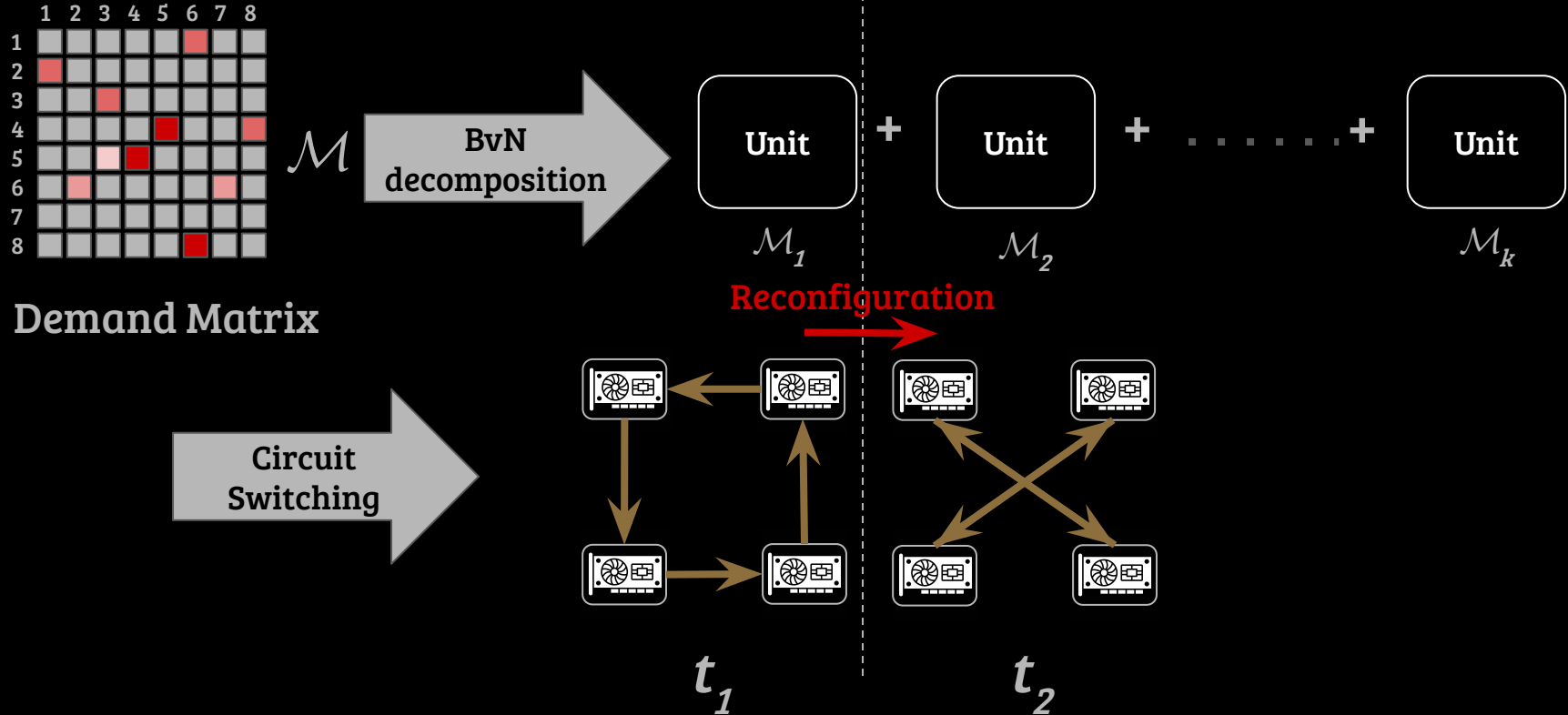
Unit

$\mathcal{M}_k$



t_1

Traditional Approach: Birkhoff Decomposition



Traditional Approach: Birkhoff Decomposition

	1	2	3	4	5	6	7	8
1								
2								
3								
4								
5								
6								
7								
8								

$\mathcal{M}$

BvN
decomposition



$\mathcal{M}_1$

+



$\mathcal{M}_2$

+

...

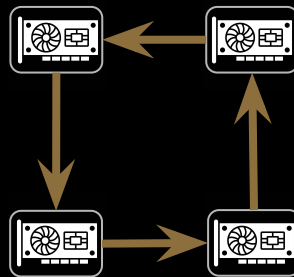
+



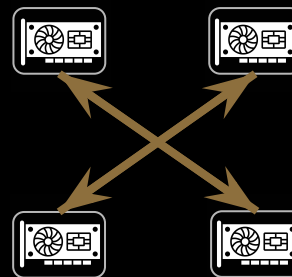
$\mathcal{M}_k$

Demand Matrix

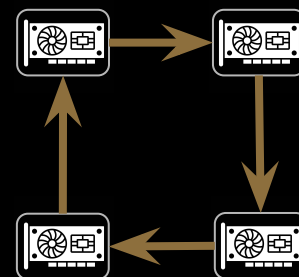
Circuit
Switching



t_1

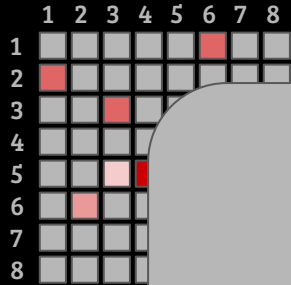


t_2



t_k

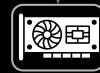
Traditional Approach: Birkhoff Decomposition



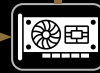
Demand

Achieves optimal completion time*

**if the reconfiguration delay is negligible*



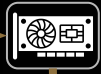
t_1



t_2



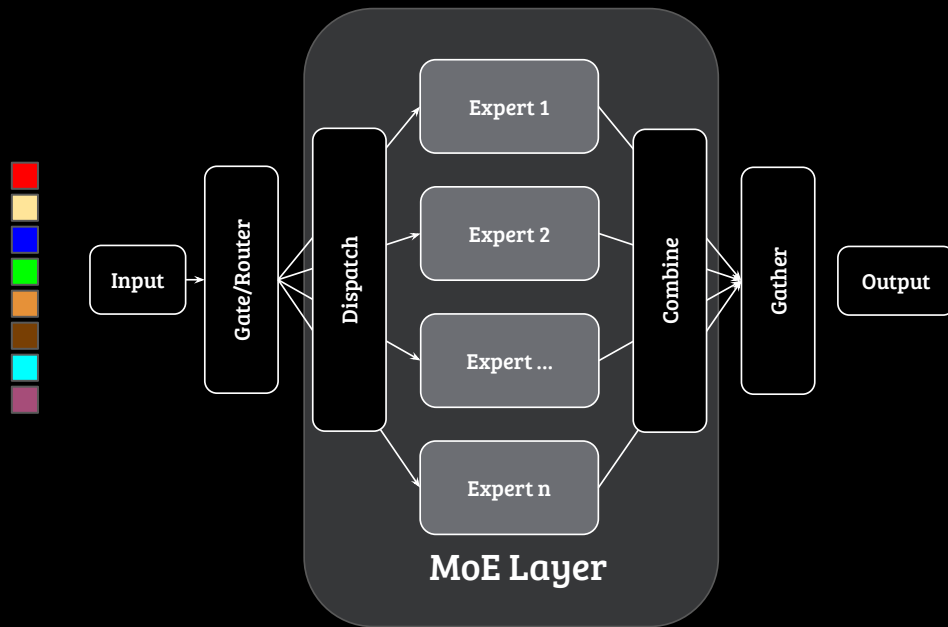
t_k



Birkhoff Decomposition is not Optimal for Makespan!

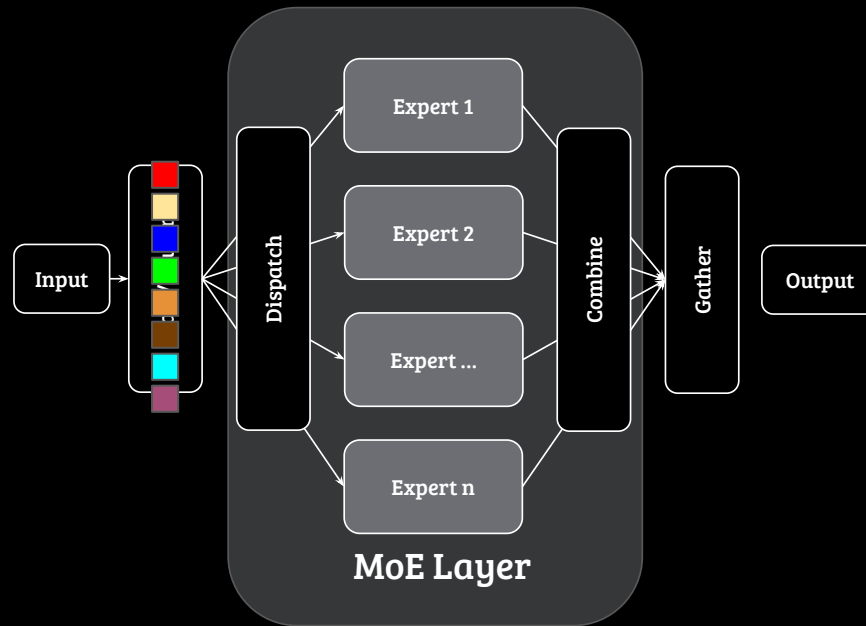
- MoE structure
- Doubly stochastic
 - Each row and column must sum to the same value
- Compute efficiency

MoE Structure



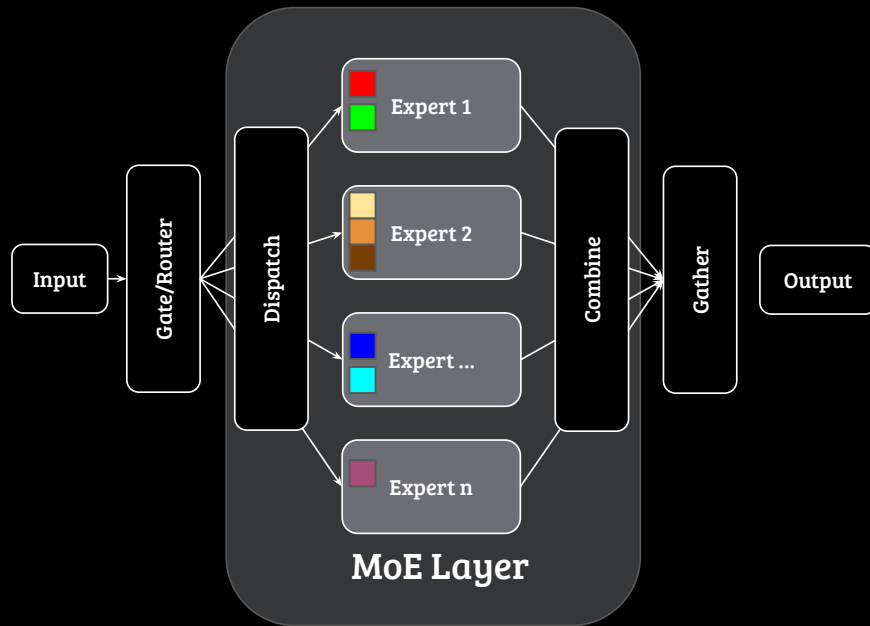
MoE Structure

Sparse computation: Router selects top-K experts



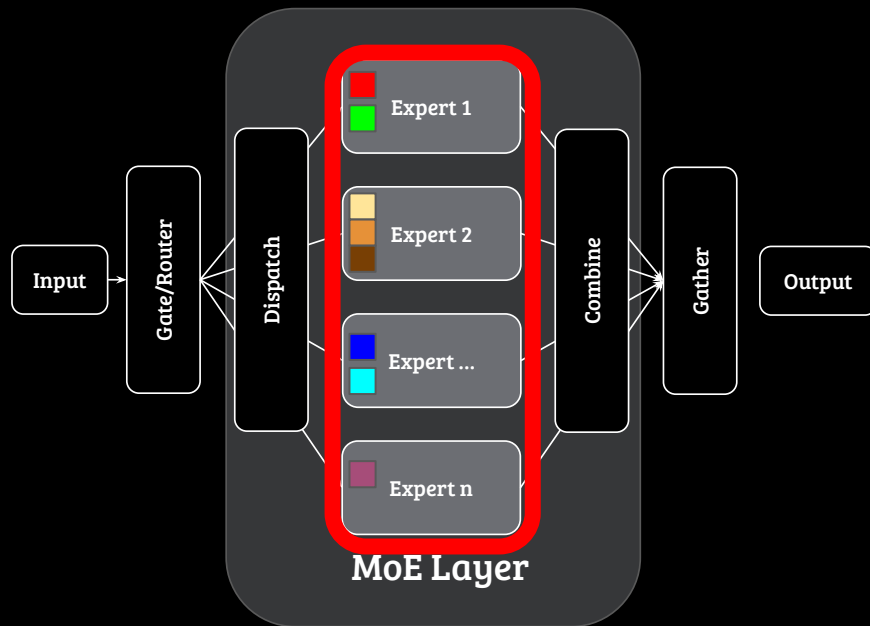
MoE Structure

Dispatch: All-to-All communication



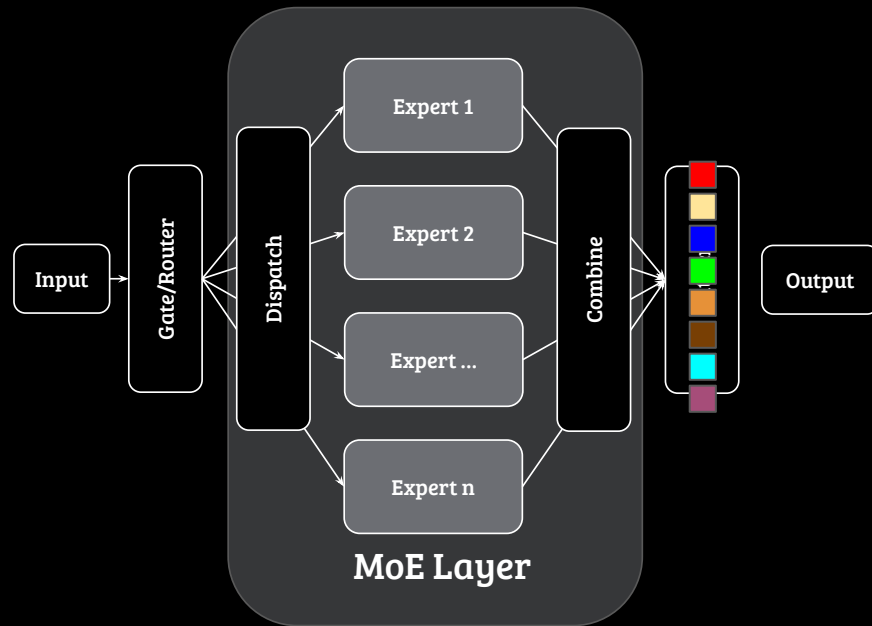
MoE Structure

Expert computation



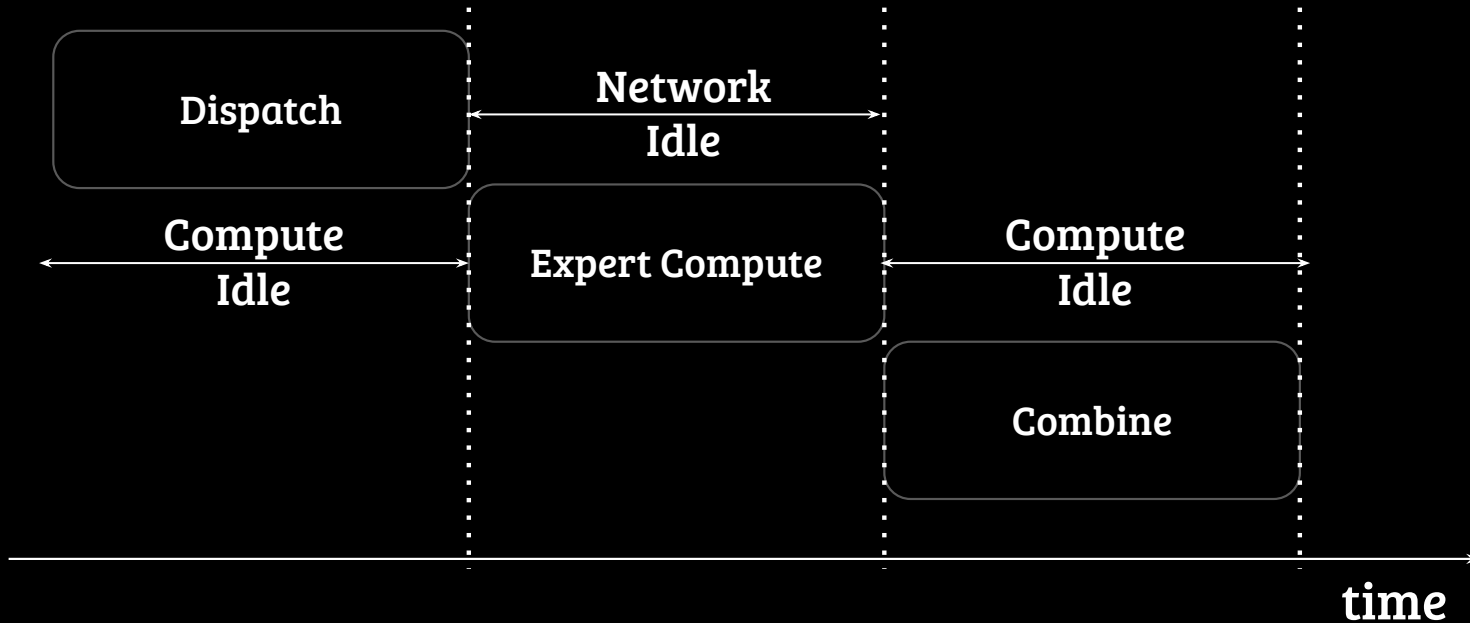
MoE Structure

Combine: All-to-All communication



MoE Structure

- Dispatch-compute-combine

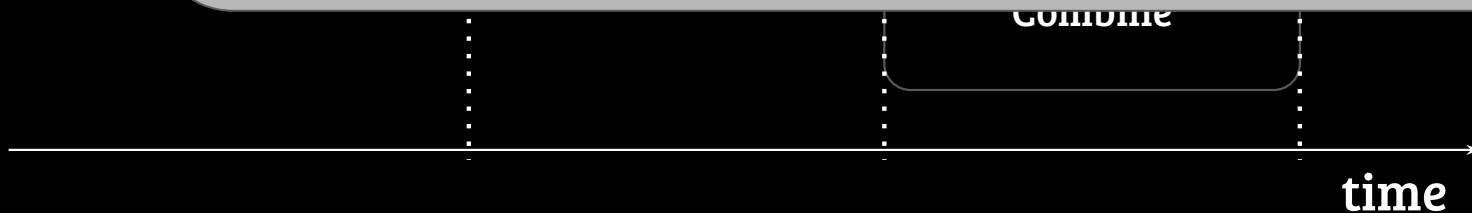


MoE Structure

- Dispatch-compute-combine

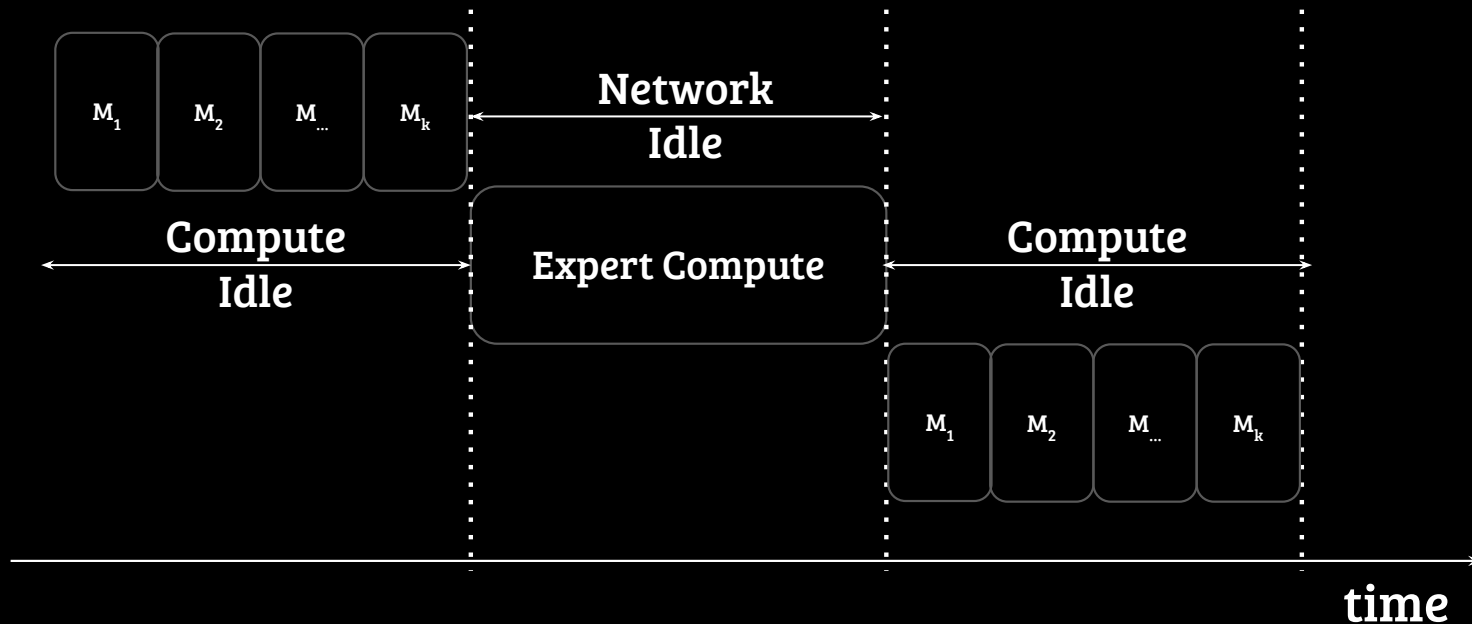
Reduce The Makespan

**Completion time of one operation is insufficient: Data Dependency*



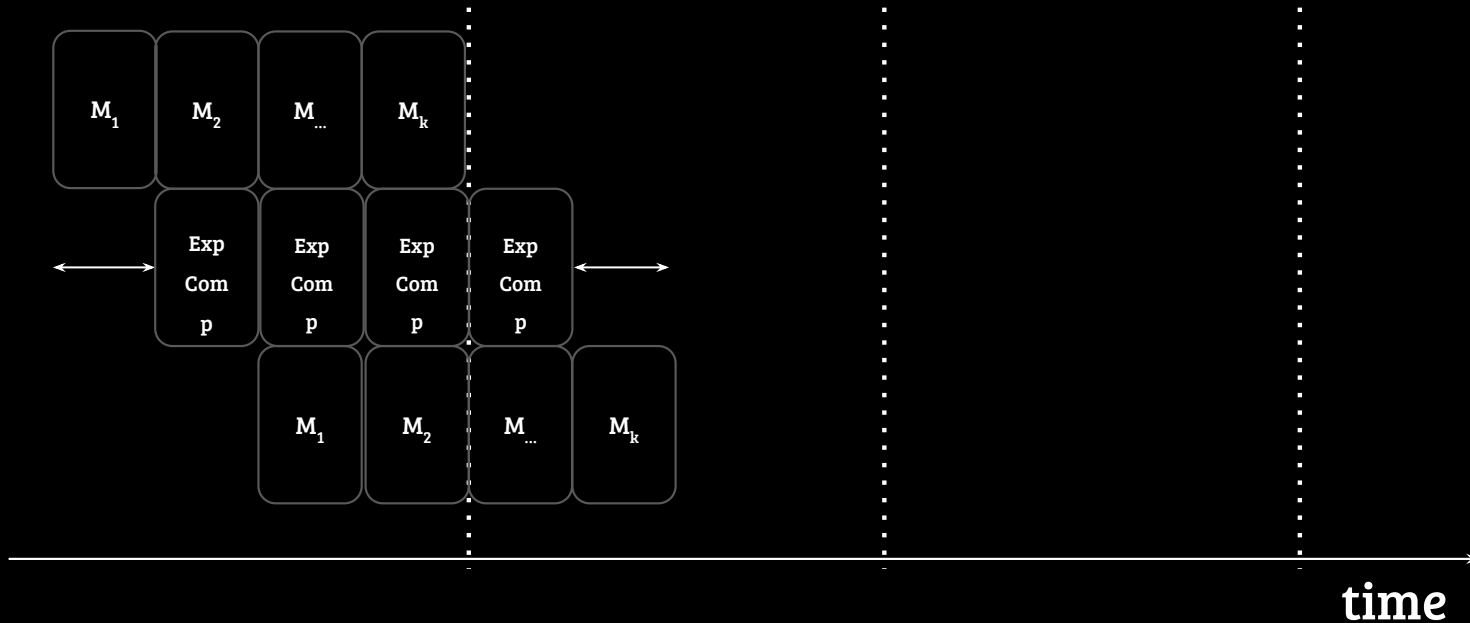
Sequential MoE Operations in OCS

- OCS decomposes the demand matrix



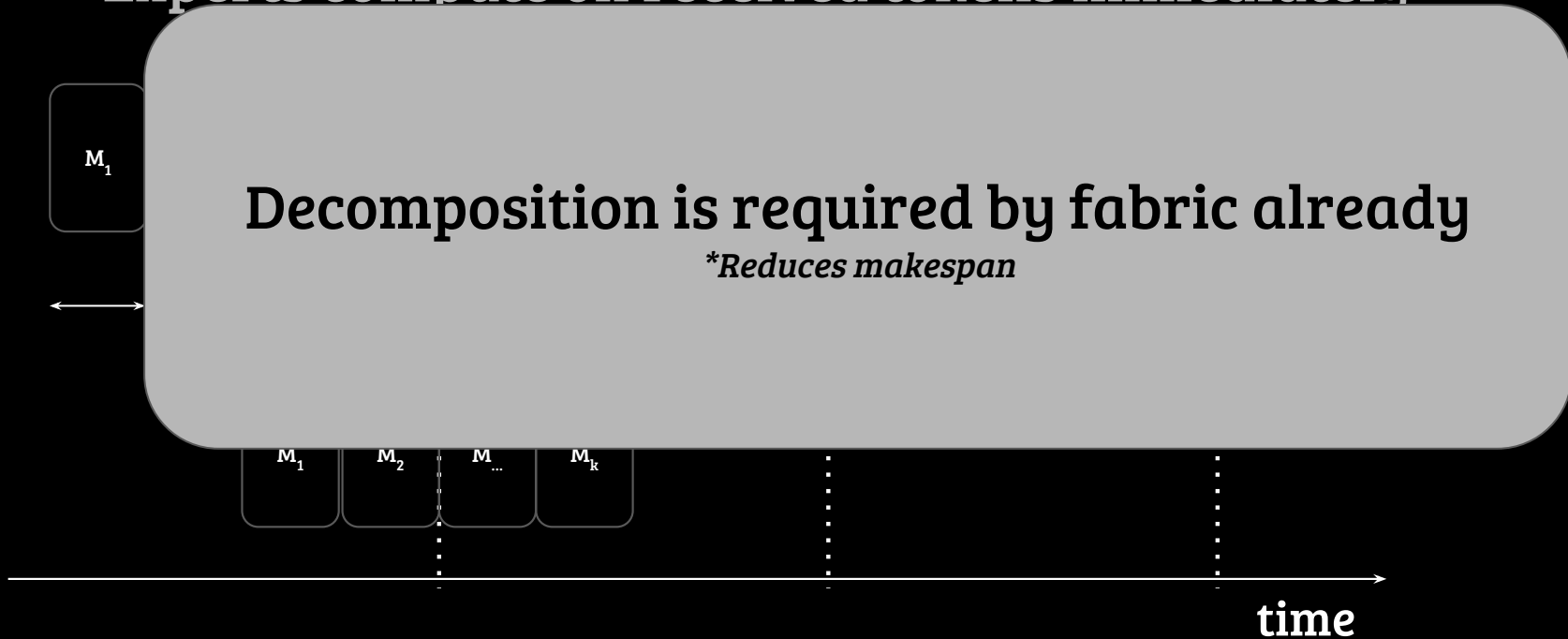
MoE Sequential Operation in OCS

- Experts compute on received tokens immediately

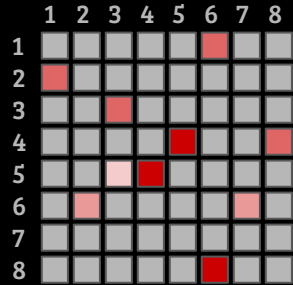


MoE Sequential Operation in OCS

- Experts compute on received tokens immediately



BvN: Overlap and Theoretical Makespan Reduction


 $\mathcal{M}$

BvN
decomposition

Unit

 $\mathcal{M}_1$

+

Unit

 $\mathcal{M}_2$

+

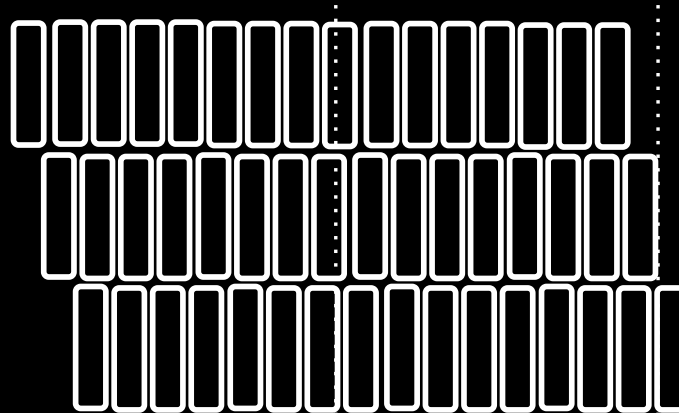
...

+

Unit

 $\mathcal{M}_k$

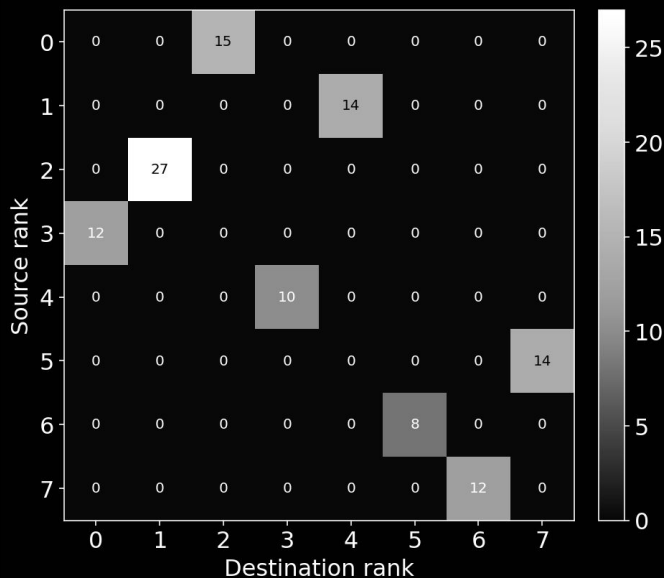
Demand Matrix



time

BvN Produces Small Batches Per Dispatch

- Many overlap opportunities
 - Up to N^2 matchings



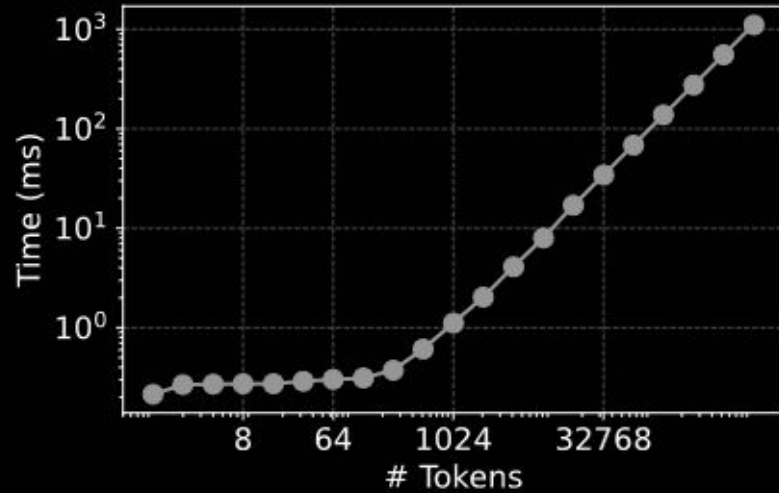
Demand Matrix

B
V
N



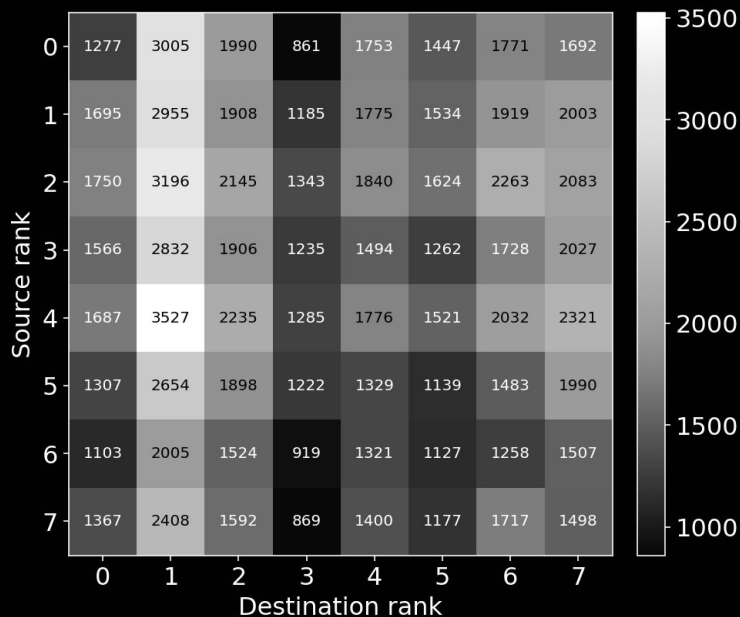
Problem 1: Compute Efficiency

Smaller batches
dominated by
compute overheads
(i.e. Kernel launches,
synchronisation)



Problem 2 : MoE All-to-All is Rarely Doubly Stochastic

- Communication imbalance
- Directly applying BvN leaves scheduling bubbles



Problem 3: BvN Produces Small Batches Per Dispatch

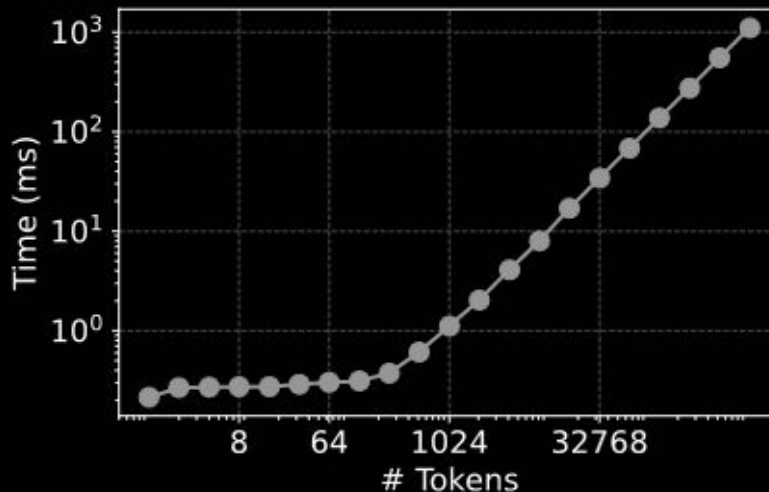
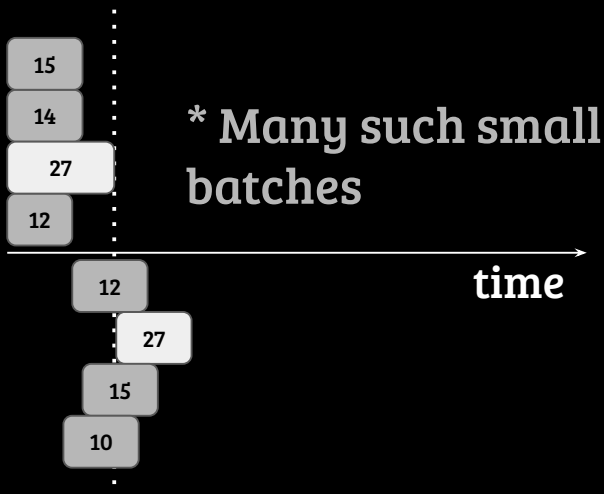
Excessive number of matchings*

**Up to $O(N^2)$ matchings). Costly if reconfiguration is not negligible*



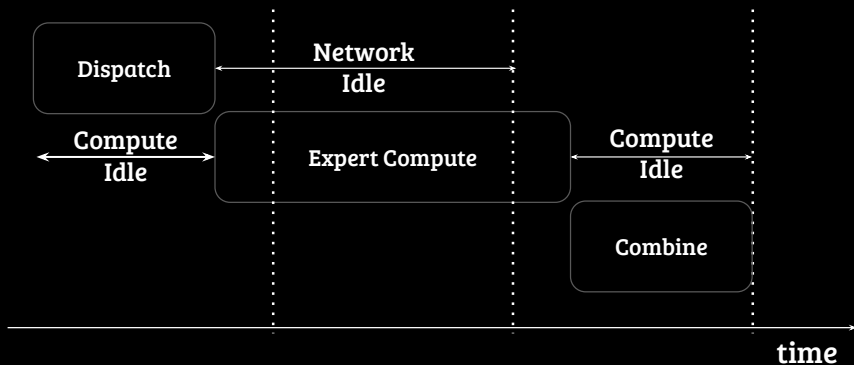
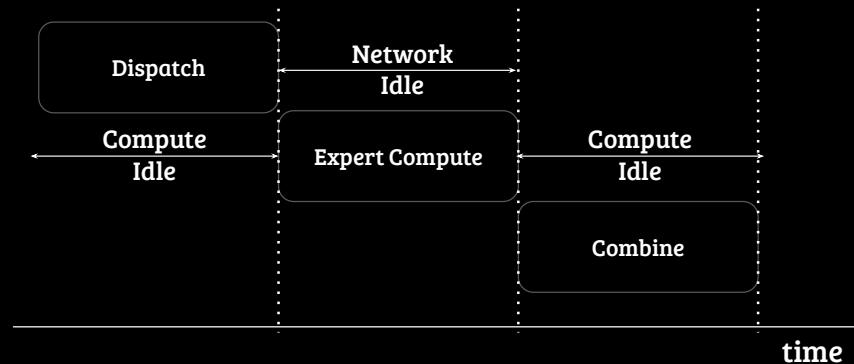
Together: BvN Decomposition is not Optimal for Makespan!

- **Fragmented Compute**
 - Wasted cycles on compute overheads
- **Can increase batch size for same compute cost**



Directive: Joint Optimize Network and Compute

- Efficiency in one operation does not equate to downstream efficiency



Decompose Differently

- Preserve large batches for compute efficiency
- Maintain beneficial properties of BvN
 - Incast/Outcast free
- Reduce number of matchings

Maxweight Decomposition

- Large Batches
- Matrix skew agnostic
- $\Theta(N-1)$ matchings

Maxweight Decomposition

	1	2	3	4	5	6	7	8
1								
2								
3								
4								
5								
6								
7								
8								

Demand Matrix

$\mathcal{M}$

MaxWeight
decomposition

Find the minimum non-zero entry and
extract a permutation (matching)



Maxweight Decomposition

	1	2	3	4	5	6	7	8
1								
2								
3								
4								
5								
6								
7								
8								

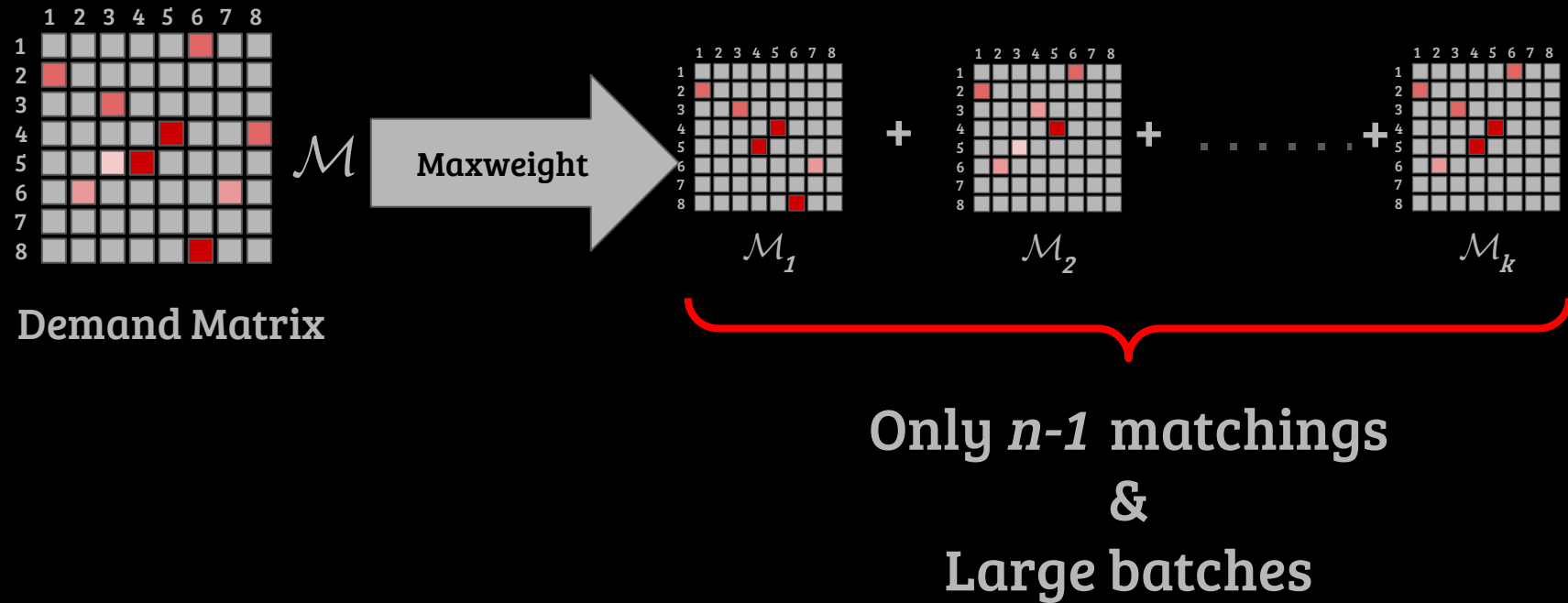
$\mathcal{M}$

MaxWeight
decomposition

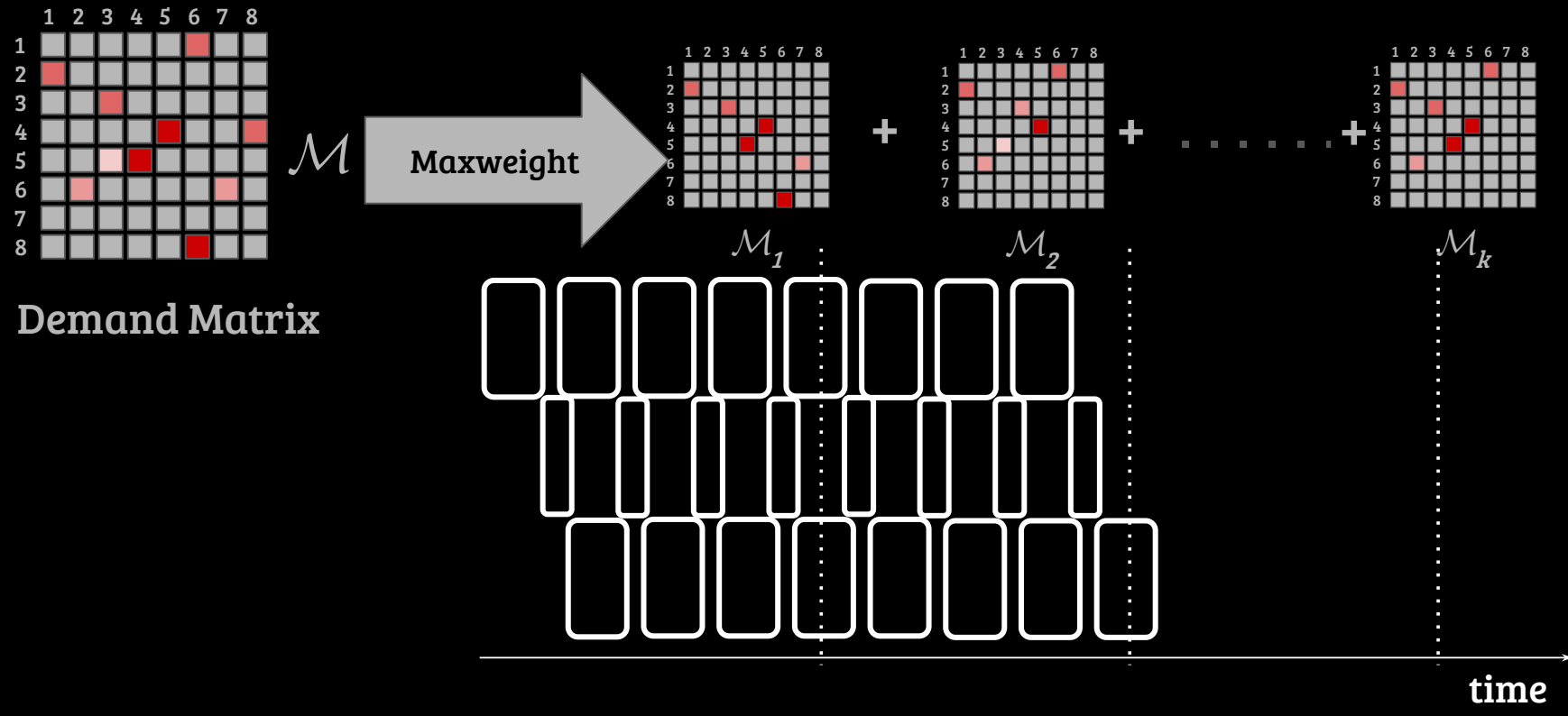
Find the maximum weight
permutation (matching)

Demand Matrix

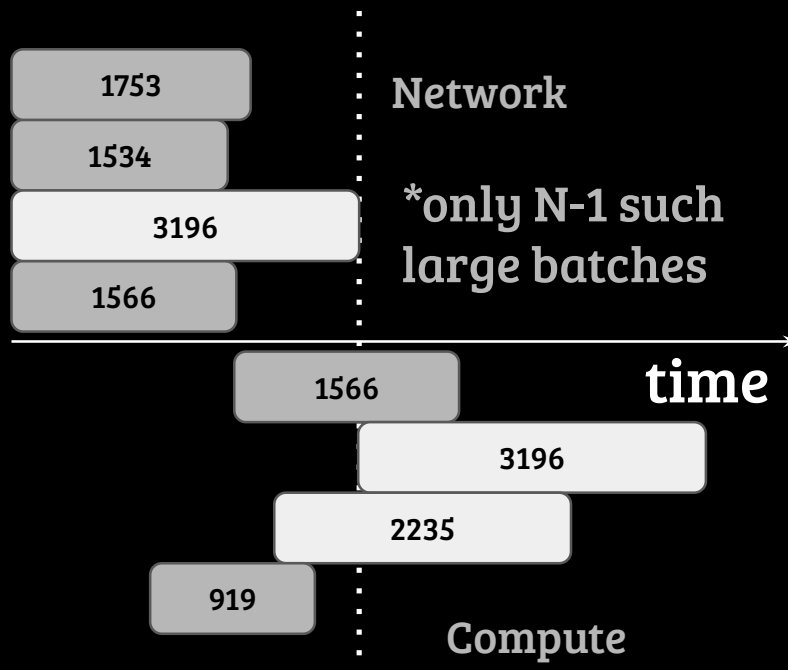
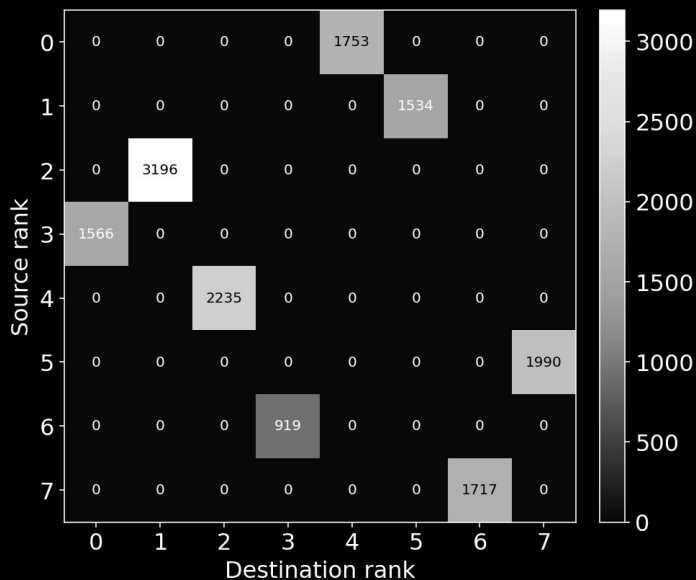
Maintain Large Batches for Compute Efficiency



Maintain Large Batches for Compute Efficiency

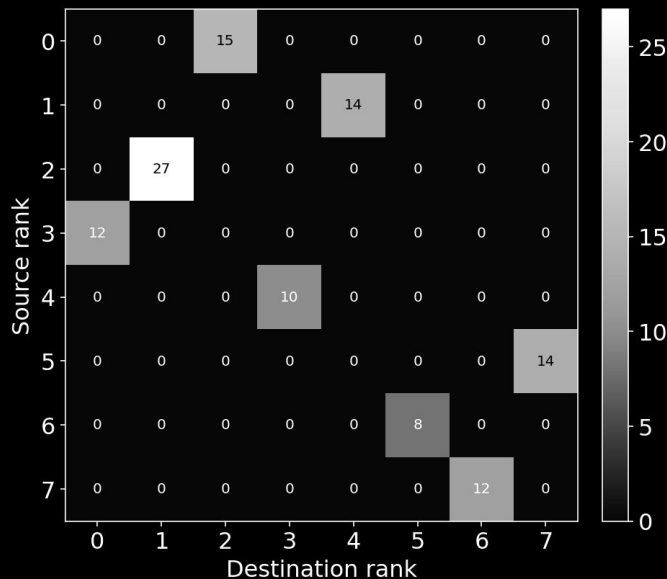


Network Gaps but Computationally Efficient

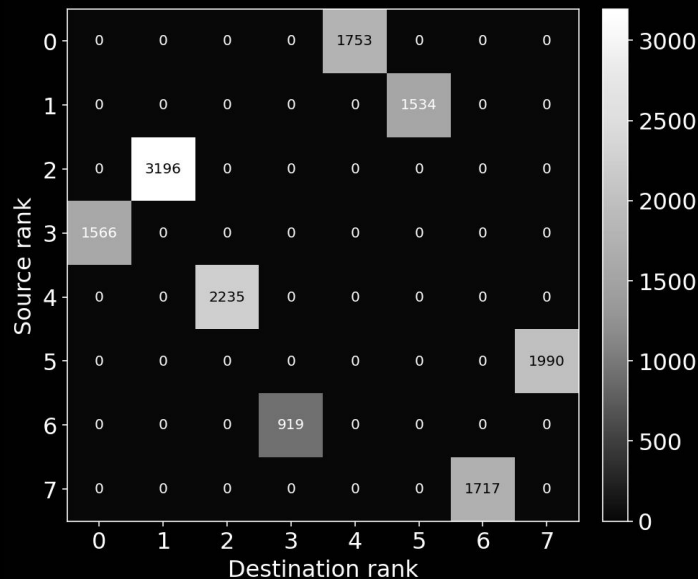


Putting it All Together: Better Compute Efficiency

- Enough compute to hide communication

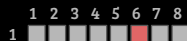
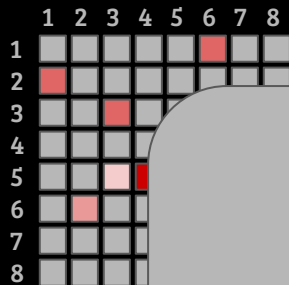


BvN



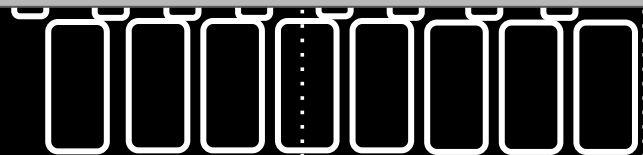
Maxweight

Putting it All Together: Efficient Compute on Large Batch



Significantly fewer number of matchings*

**N-1 matchings. Preserves large chunks for compute*



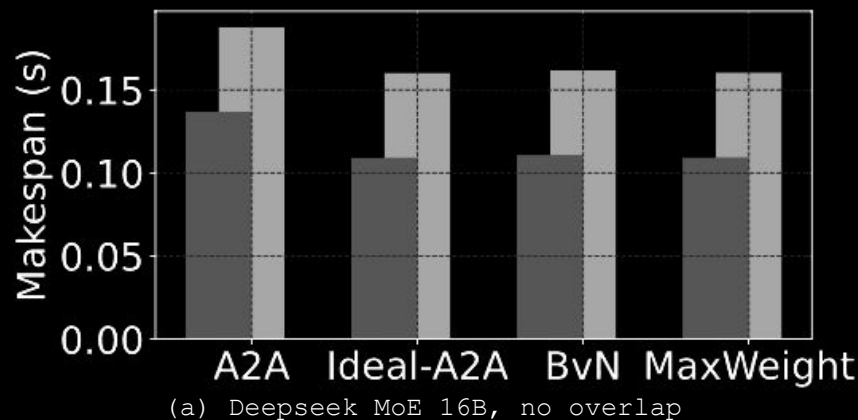
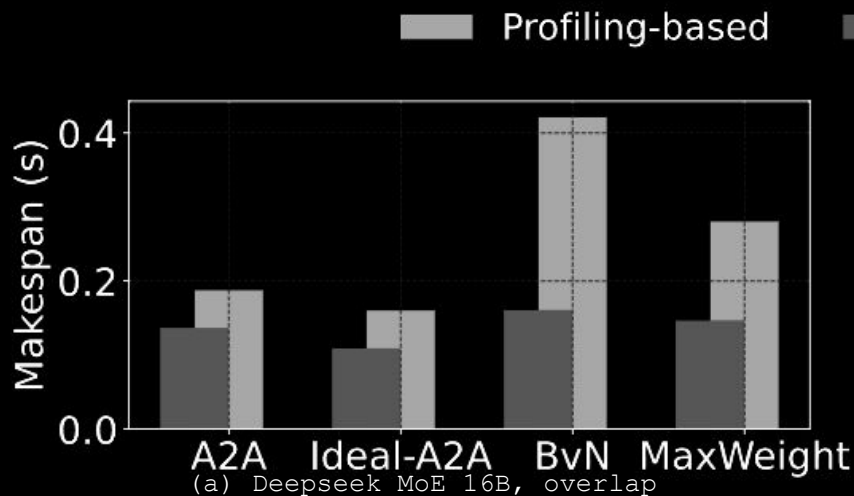
time

Evaluation

- Real MoE traffic matrices
- 8 RTX 6000 PRO
- Sequential vs Overlap
- Ideal All-to-all (congestion free solved in Gurobi)

Together: Severe Compute Inefficiency

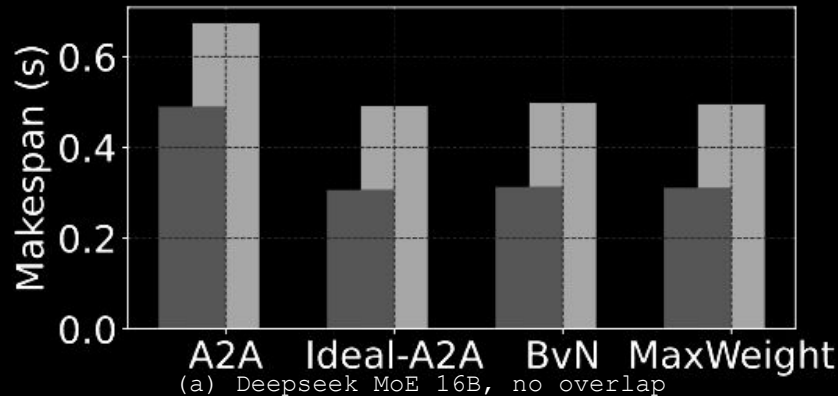
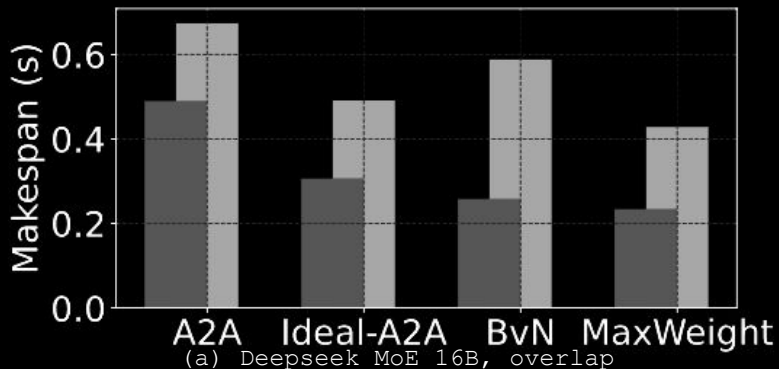
- MMLU Dataset: Naturally small prompts



Together: Efficient Compute on Large Batch

- **SpeedBench Dataset: Naturally large prompts (~2K tokens per prompt)**

Profiling-based
 Linear cost model



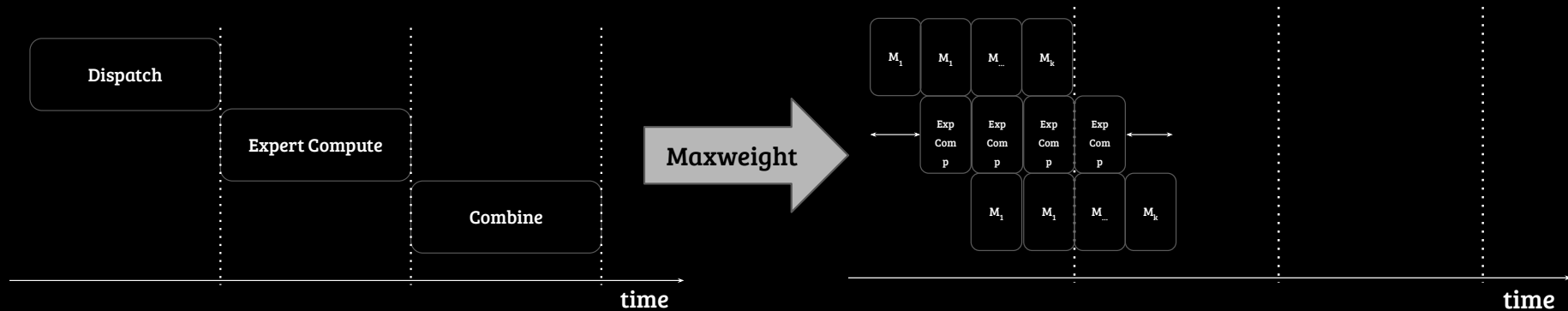
Takeaway: Jointly Optimize Network and Compute

- Makespan vs Completion Time
 - Data dependency
 - **Dispatch-Compute-Combine**



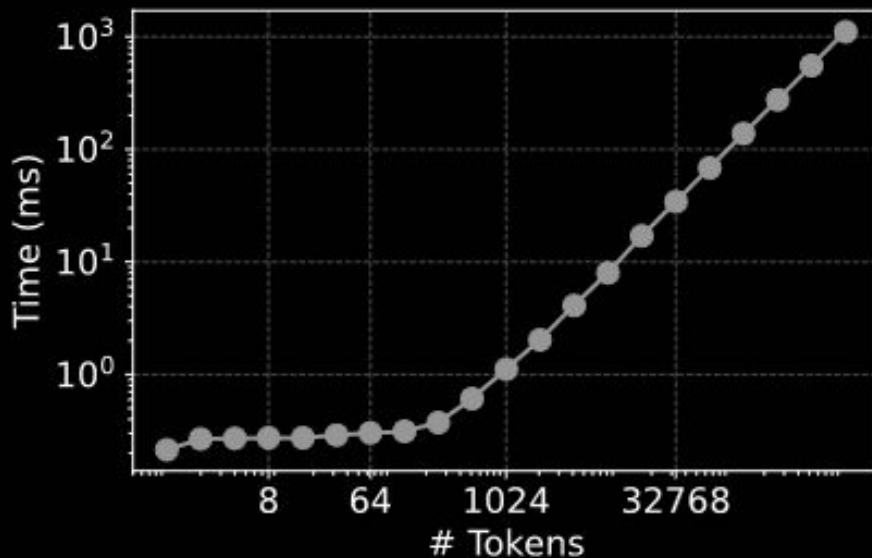
Takeaway: Overlap Opportunities

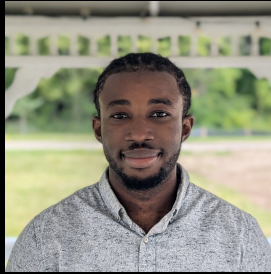
- Matrix decomposition strategies in OCS naturally provides overlap
 - More matchings, more overlap opportunities



Finally: Decomposition Quality

- Not just number of matchings
 - Structure of decomposition
 - Transmission Imbalance
 - **Compute Fragmentation**





Eliezer Amponsah
eampons@purdue.edu



Vamsi Addanki
vaddank@purdue.edu

Thank You